GSE Geometry Milestone Study Guide Name Key S: ____

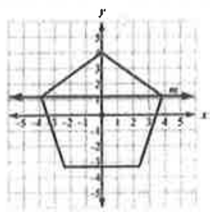
Unit 1 - Transformations

Isometry: A distance preserving map of a geometric figure to another location using a reflection, rotation or translation.

Rotation: Rules are in terms of counter clockwise $R_{90} = (-y, x)$ $R_{180} = (-x, -y)$ $R_{270} = (y, -x)$

Reflection: A transformation about a line that acts as a mirror; x = 0 is a vertical LoR & y = 0 is a horizontal LoR.

1) A regular pentagon is centered about the origin and has a vertex at (0, 4). Which transformation maps the pentagon onto itself?
   A. a reflection across line m.
   B. a reflection across the x-axis.
   C. a clockwise rotation of 100° about the origin.
   D. a clockwise rotation of 144° about the origin.



$\frac{360}{5} = 72°$

$72 \times 2 = 144°$

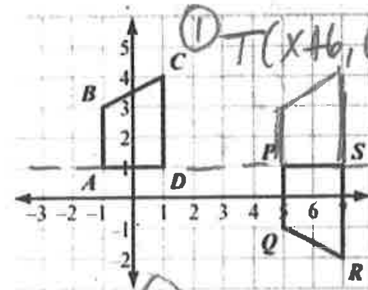
2) Is a dilation an isometry? Why?

no, it does not presene size

3) Describe transformations that map ABCD to PQRS.

① T(x+6, 0)

y = 1



② Reflection y = 1

4) The point $Y(-1, 7)$ has been rotated 90^0 counter clockwise around the origin. Where is the new location of point Y?

$(-1, 7) \rightarrow (-y, x) \rightarrow (-7, -1)$

y is changing, not x

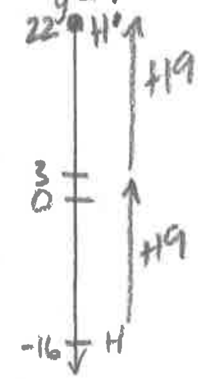
5) **Reflection:** About $y = 3$, gives what new vertices? $H(12, -16), J(-9, -3), K(-17, 12), L(13, 11)$

+19 +6

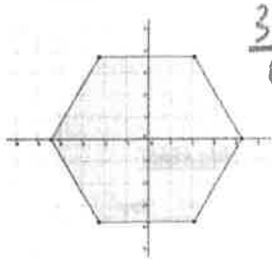
H' = (12, 22) J' = (-9, 9)

K' = (-17, -6) L' = (13, -5)

-9 -8

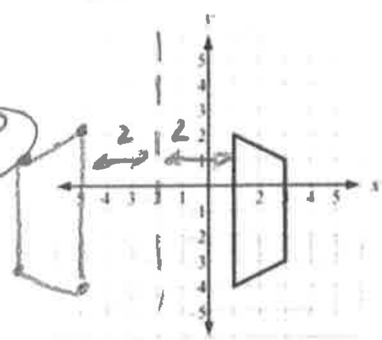


6) **Degrees of Rotation:** What is the minimum degrees of rotation to map the regular hexagon onto itself?



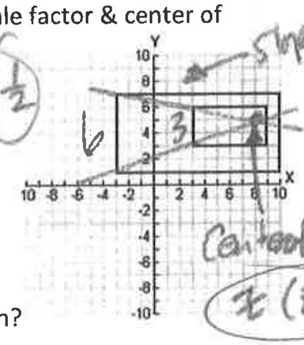
$\frac{360}{6} = 60°$

8) **Reflection:** About $x = -2$.



9) **Dilation:** A large rectangle is dilated to a smaller one. What is the scale factor & center of dilation?

$SF = \frac{5}{8} = \frac{3}{6} = \frac{1}{2}$



(8, 5)

7) Give three multiples of rotations that map the hexagon onto itself?

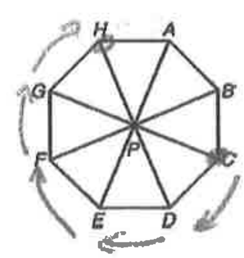
120, 180, 240

10) If the result of $(x, y) \rightarrow (x - 1, y + 2)$ *is* $A'(-5, 2)$, what is the **pre-image**, or A?

$(x - 1 = -5, y + 2 = 2)$
$+1$ -2

$(-4, 0)$

11) What ***clockwise*** rotation of the octagon at right about point P maps point C to point H?



$\frac{360}{8} = 45$

45×5

225°

5 rotations

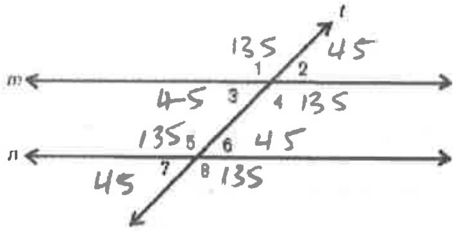
Unit 2 – Triangle Similarity & Congruence

1) Angle 5 is alternate interior to angle? ∠4

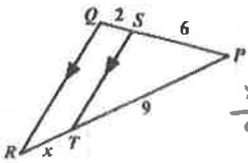
2) Angle 7 is corresponding to angle? ∠3

3) Angle 8 is vertical to angle? ∠5

4) If angle 1 equals 135 degrees, fill in all remaining angles.



5) Triangle Proportionality: Find x.



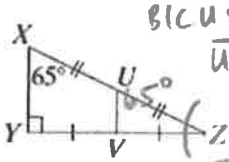
$\frac{x}{9} = \frac{2}{6}$

$\cancel{6x = 18}$

x=2

6) Similar Triangles: What is the reflexive angle? Is XZY similar to UZV? If so, how?

B/C U & V ARE MIDPOINTS, $\overline{UV} \parallel \overline{XY}$.

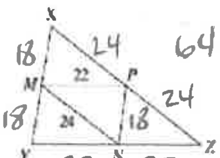


∠Z Reflexive

AA~

7) Midsegment: If M, N, and P are midpoints & perimeter of MPN = 64, find the length of all segments.

64−22−24=18



64 x 2 = 128 units

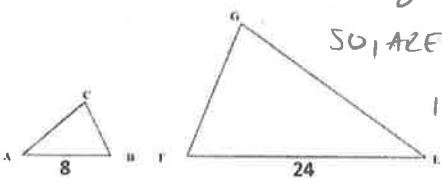
$\checkmark$ 36+48+44 =128

8) The sketch below shows 2 similar △'s, ABC and EFG. ABC has an area of 12 units, and it's base, AB, is 8 units long. The base of DEF is 24 units. What is the area of DEF?

SF = $\frac{24}{8} = (3)^2 = 9$

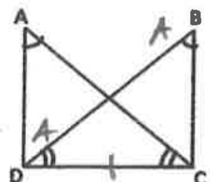
SO, AREA OF BIG △ IS

12 x 9 = 108 units²



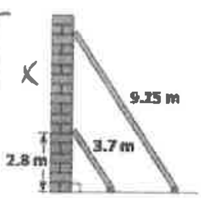
9) *Given:* ∡ *DAC* ≅ ∡*CBD*, ∡ *BDC* ≅ *ACD*

Prove: $\overline{AC} \cong \overline{BD}$



Statement	Reason
1. ∠DAC ≅ ∠CBD	1. Given
2. ∠BDC ≅ ∠ACD	2. GIVEN
3. DC ≅ DC	3. Reflexive Property
4. △ CDA ≅ △ DCB	4. AAS
5. AC ≅ BD	5. CPCTC

10) What is the height between the tops of the two ladders?



2.8 x 2.5

$\frac{B}{S} = \frac{9.25}{3.7} = 2.5$ SF

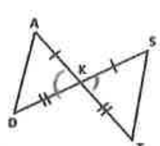
Whole wall HT = 2.8 x 2.5 = 7

x = 7 − 2.8 = 4.2

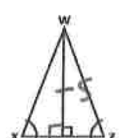
Triangle congruency: ***SSS, SAS, ASA, AAS, HL****. Remember SSA / ASS can't prove congruency.* ***You can't double skip!***

11) △ *RAC* ≅ △RTO by AAS or ASA

12) △ *KAD* ≅ △KST by SAS

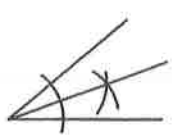


13) △ *XYW* ≅ △ZYW by AAS

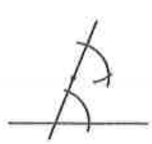


14) Geometric Constructions – Identify each partial or full construction

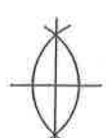
copy ∡



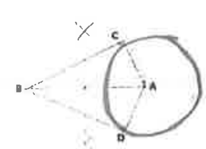
Bisect ∡



∥ lines



⊥ bisector



tangent(s)

Unit 3 - Right Triangle Trigonometry

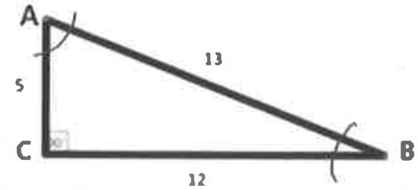
Key Concepts

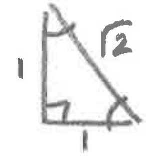
A missing side can be found using $\sin\theta = \left(\frac{o}{h}\right)$, $\cos\theta = \left(\frac{a}{h}\right)$, *or* $\tan\theta\left(\frac{o}{a}\right)$ *when you know an angle and one side of a right triangle.*

An angle θ *can be found by using one of* $\sin^{-1}\left(\frac{o}{h}\right)$, $\cos^{-1}\left(\frac{a}{h}\right)$, *or* $\tan^{-1}\left(\frac{o}{a}\right)$ *when two sides are known of a right triange.*

$Sin\ A = Cos\ B$ *when angles A and B are complementary in a right triangle:* $Sin\ A = Cos\ (90 - A)$

Using the diagram for 1-9. First, find each trig ratio.





1) Sin A = 12/13   *4) Sin B =* 5/13

2) Cos A = 5/13   *5) Cos B =* 12/13

3) Tan A = 12/5   *6) Tan B =* 5/12

see above

= cos B

7) True or False: $Sin\ A = Cos\ (90 - A)$

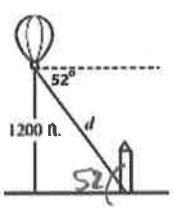
Explain. 90 − A = B

8) Tan A & Tan C are ~~reciprocals~~

9) In a 45 − 45 − 90 triangle, the ratio of Sin A = cos A

Sin A = $\frac{1}{\sqrt{2}}$ cos A = $\frac{1}{\sqrt{2}}$

10) **Angle of Depression & Elevation**: If the AoD is 52 degrees, solve for d.



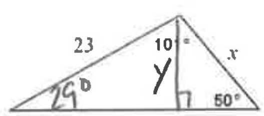
$\sin 52 = \frac{1200}{d}$

$d = \frac{1200}{\sin 52} = 1522.8$ ft

11) **Drop an altitude**: Solve for x.

$\sin 29 = \frac{y}{23}$ $y = 23 \cdot \sin 29$

$= 11.2$



$\sin 50 = \frac{11.2}{x}$

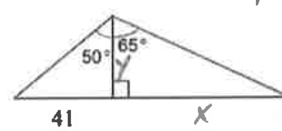
$x = 11.2 / \sin 50 = 14.6$

12) **Area:** Solve for total area.

$\tan 50 = \frac{41}{y}$ $y = 34.4$

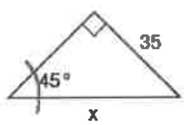
$\tan 65 = \frac{x}{34.4}$

$x = 73.8$



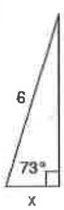
Area = $\frac{\frac{41+73.8}{2} \cdot 34.4}{2} = 987.3$ u²

13) **Regular Trig: Find the missing side.**



$\sin 45 = \frac{35}{x}$ $x = 49.5$

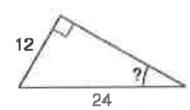
14) **Regular Trig: Side**



$\cos 73 = \frac{x}{6}$

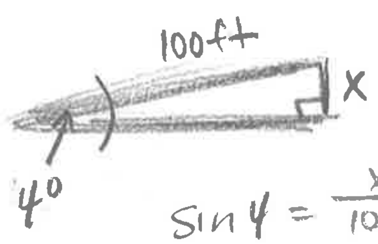
1.75

15) **Inverse Trig:** Find the angle.



$\sin^{-1}(12/24) = 30°$

16) A road ascends a hill at an angle of 4° for every 100 feet of road, how many feet does the road ascend? **Draw a diagram.**



$\sin 4 = \frac{x}{100}$

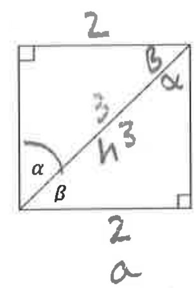
$x = 100 \cdot \sin 4$

$= 6.97$ ft

17) In this figure, two right angles and two adjacent angles, α & β, are shown. If $\sin(\alpha) = \frac{2}{3}$, what is the value of $\cos(\beta)$?

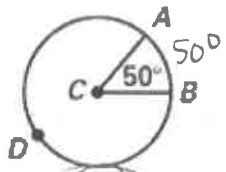
$\cos\beta = \frac{a}{h}$

$= \frac{2}{3}$



Unit 4 – Circles, Angles & Segments

Central ∡ = Intercepted arc

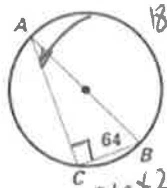


50°

AB = ? 50°

Diameter of Inscribed ∡

$m\widehat{BC}$ $90-64=26$



180

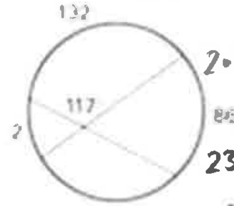
26×2

52°

Interior ∡ = ½ Sum ⌒

Find arcs 1 & 2.

$117 = \frac{x+132}{2}$



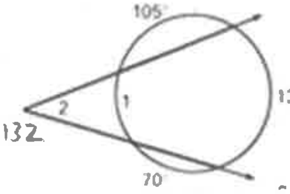
$2 \cdot 117 = x+132$

$234 = x+132$

$x = 234-132$

$x = 102^\circ$

Exterior ∡ = ½ diff ⌒



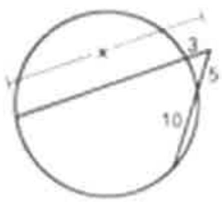
Find arc 1 & angle 2/

$360 - 105 - 130 - 70 = 55$

$= \frac{130-55}{2}$

$= 37.5^\circ$

Outside(whole) =



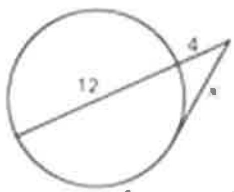
Find x.

$3(x) = 5(15)$

$3x = 75$

$x = 25$

Tangent is outside & whole

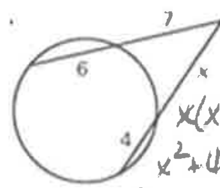


Find x. $x^2 = 4(16)$

$x^2 = 64$

$x = 8$

Missing Outside = Quadratic



$x(x+4) = 7(13)$

$x^2 + 4x = 91$

Find x. $x^2 + 4x - 91 = 0$

$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-91)}}{2}$

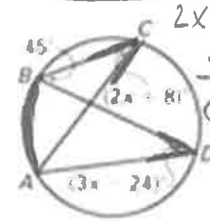
$x = 7.75$

Shared Inscribed ∡'s are ≅

$2x + 8 = 3x - 24$

$+24 \quad -2x$

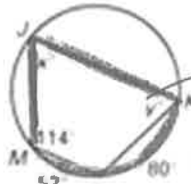
$32 = x$



$2(32) + 8 = 72^\circ$

Find x & angle BCA.

Inscribed Quad is Supplementary



$180 - 114$

66°

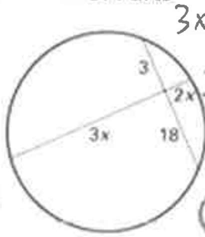
$\angle x = \frac{52+80}{2}$

$= 66^\circ$

Find x & y.

Product of Parts

$3x(2x) = 3(18)$



$\frac{6x^2}{6} = \frac{54}{6}$

$x^2 = 9$

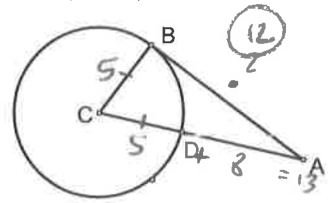
$x = 3$

Find x and lengths of chords.

$3+18 = 21$ $9+6 = 15$

Point of Tangency: AB is tangent to circle C at B.

AD=8, CB=5, AB = ?



$= 13$

12

$AB = \sqrt{(13)^2 - (5)^2}$ $= 12$

Arc Length & Sector Area

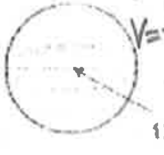
An apple pie has a diameter of 9 in. The pie is cut into 6 equal pieces. What is the area and arc length of 4 pieces of pie?

$r = \frac{9}{2} = 4.5$

$A_L = \frac{2\pi(4.5)4}{6}$ $A_{OS} = \frac{\pi(4.5)^2 4}{6}$

$= 18.85 in$ $= 42.41 in^2$

Sphere Volume:



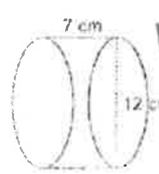
$V = \frac{4}{3}\pi(6.4)^3$

$=$

$\frac{12.8}{2} = 6.4$

$= 1098.1 u^2$

Cylinder Volume:



$/2$

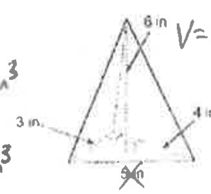
$V = \pi(6)^2 \cdot 7$

$252\pi \ cm^3$

or

$791.7 cm^3$

RT Triangle Pyramid volume:



$V = \frac{\frac{3 \cdot 4}{2} \cdot 6}{3}$

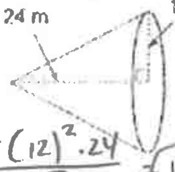
$= \frac{6 \cdot 6}{3}$

$= 12 in^3$

Cavalieri's Principle: Can the cylinder and RTΔ pyramid at left have the same volume if they have the same height?

no, b/c the pyramid slants to a single pt.

Cone Volume:

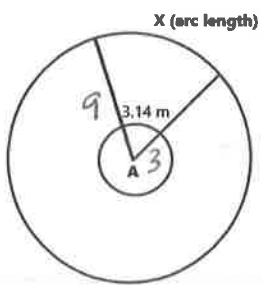


or $3619.1 m^3$

$V = \frac{\pi(12)^2 \cdot 24}{3}$ $= 1152\pi \ m^3$

Composite Volume:

At right, the radius of the smaller circle = 3 m while the radius of the larger circle is 9 m. The arc length intercepted by the small circle is 3.14 m. what is the arc length of the larger circle? s = AL, r = radius



$\frac{s}{r} = \frac{s}{r}$

$\frac{9}{3} = 3 = SF$

$3 \times 3.14 = 9.42 m$

Identify 2D shapes as 3D Objects: If a circle is rotated, what 3D shape will result?

Sphere

Unit 5 – Algebraic Connections with Geometry

Key Concepts

Distance: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, and you can always draw a right triangle on a graph to find Δx *and* Δy.

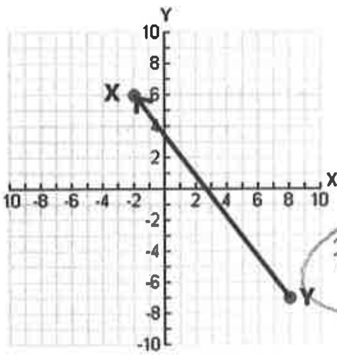
Midpoint: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Point Partitioning a Line Segment: $(x, y) = (x_1 + \frac{A}{A+B}(\Delta x), y_1 + \frac{A}{A+B}(\Delta y))$

Standard Form of a Circle: $(x-h)^2 + (y-h)^2 = r^2$, **where the number on the right is ALWAYS squared.**

A parallelogram and rhombus have diagonals that bisect. A rectangle and square have diagonals that are congruent.

1) **Partitioning:** Find Point Z that partitions the directed line segment $\overline{YX}$ in a ratio of $\frac{5}{3}$, $X(-2, 6)$ *and* $Y(8, -7)$. Graph.



$x = 8 + \frac{5}{8}(-10)$

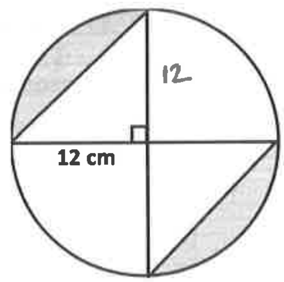
$y = -7 + \frac{5}{8}(13)$

$Z\left(\frac{7}{4}, \frac{9}{8}\right)$

$(1.7, 1.125)$

2) **Sector Area:** 2 diagonals of a circle are shown, and the radius is 12 cm. What is the area of the shaded regions?

2 sectors of 4



$A_{OS} = \frac{\pi(12)^2 \cdot 2}{4} = 226.2$

2 Δ's

$-\ A_{OT}\quad \frac{12 \cdot 12}{2} \cdot 2 = 144$

$82.2\,cm^2$

3) **Completing the Square:** Put into standard form, find center & radius. $4x^2 + 4y^2 - 24x + 48y + 13 = 0$

$x^2 - 6x + \underline{9}\ \ y^2 + 12y + \underline{36} = -3.25$

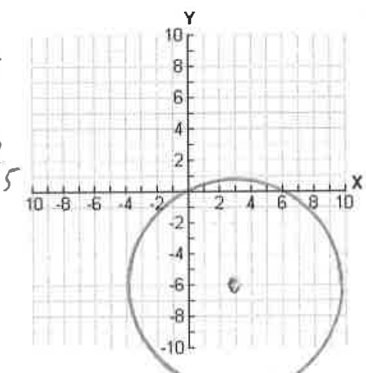
$\frac{-6}{2} = (-3)^2 \qquad \frac{12}{2} = (6)^2 \qquad \begin{array}{r} +9 \\ +36 \\ \hline 41.75 \end{array}$

$(x-3)^2 + (y+6)^2 = 41.75$

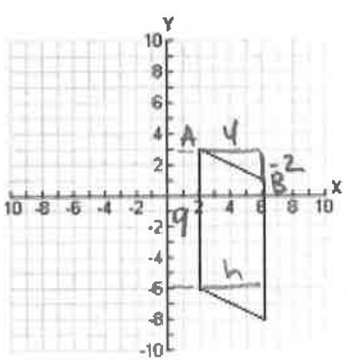
$C: (3, -6)\ r = \sqrt{41.75}$

6.5

4) **Graphing Circles:** Now graph the circle from #3.



5) **Distance Formula:** Find the perimeter and area. b·h



$P = 9 + 9 + 2\sqrt{5} + 2\sqrt{5}$

$d_{AB} = \sqrt{(4)^2 + (-2)^2}$

$= \sqrt{20}$

$= 2\sqrt{5}$

$P = 18 + 4\sqrt{5}$ 26.94

Area $= 9 \times 4 = 36u^2$

6) **Circle Properties:** Which point shown below lies on a circle with a center of $(3, -9)$ and a radius of $\sqrt{34}$?

$(6, -3)$ or $(1, -2)$ or $(1, -4)$ or $(0, -4)$ $\quad (x-3)^2 + (y+9)^2 = 34$

r^2

$(0-3)^2 + (-4+9)^2 =$

$(-3)^2 + (5)^2 = 9 + 25 = 34$ ✓

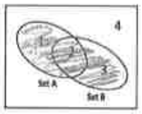
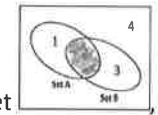
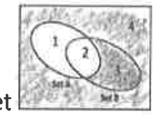
7) Find the midpoint: $(-10, -5)$ & $(13, 8)$

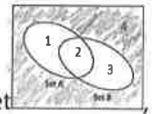
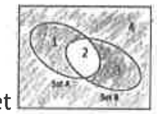
$MP = \left(\frac{-10+13}{2}, \frac{-5+8}{2}\right)$

$= \left(\frac{3}{2}, \frac{3}{2}\right)$

Unit 6 - Probability

Key Concepts

Given $A \cup B$ shade the set , *Given* $A \cap B$ shade the set , *Given* $\bar{A}$ *or* A' shade the set 

Given $(A \cup B)'$ shade the set , *Given* $(A \cap B)'$ shade the set 

Addition Rule (aka mutually exclusive): $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Multiplication Rule for Independent Events: $P(A \cap B) = P(A) * P(B)$

Conditional Probability: $P(A \cap B) = P(A) * P(B/A)$ **or** $\boxed{P(B|A) = \frac{P(A \cap B)}{P(A)}}$

Independent Events do not affect one another while Dependent Events do and means non-replacement.

1) Find the probability that a randomly selected student will be a junior, given that the student owns a car.

$\frac{P(J \cap CAR)}{P(CAR)} = \frac{6}{18} = \frac{1}{3}$

2) Find the probability that a randomly selected student will own a car, given that the student is a senior.

$\frac{P(CAR \cap S)}{P(S)} = \frac{12}{20} = \frac{3}{5}$

Car Ownership by Grade

	Owns a Car	Does Not Own a Car	
Junior	6	10	
Senior	12	8	20
TOTAL	18	18	

3) For two events B and C, it is known that $P(C|B) = 0.65$ and $P(C \cap B) = .43$. Find P(B).

$.65 = \frac{.43}{P(B)} = \frac{.43}{.65} = .662$ or 66.2%

4) For two events X and Y, it is known that $P(X) = \frac{5}{24}$ and $P(X \cap Y) = \frac{1}{8}$. Find $P(Y|X)$.

$\frac{\frac{1}{8}}{\frac{5}{24}} = \frac{1}{8} \cdot \frac{24}{5} = \frac{3}{5}$

4) A sock drawer contains 5 pairs of each color socks: white, green and blue. What is the probability of randomly selecting a pair of blue socks, replacing it, and then randomly selecting a pair of white socks?

$P(B \cap W) = \frac{5}{15} \cdot \frac{5}{15} = \frac{25}{225} = \frac{1}{9}$

5) Randy has 8 pennies, 3 nickels, and 5 dimes in his pocket. If he randomly chooses 2 coins, what is the probability that they are both pennies if he doesn't replace the first one?

$\frac{8}{16} \cdot \frac{7}{15} = \frac{56}{240} = \frac{7}{30}$

6) Using the letters in the state MISSISSIPPI. Find the probability of picking an **S** and then a **P** without replacement.

$\frac{4}{11} \cdot \frac{2}{10} = \frac{8}{110} = \frac{4}{55}$

7) Determine if the following events are independent.

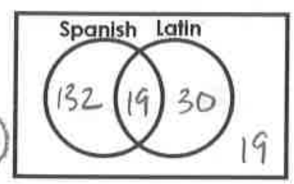
$P(A) = \frac{3}{4}, P(B) = \frac{5}{6}, P(A \cap B) = \frac{5}{8}$.

yes, A & B ARE independent

$P(A) \cdot P(B)$

$\frac{3}{4} \cdot \frac{5}{6} = \frac{15}{24} = \frac{5}{8} = \frac{5}{8}$ ✓

A guidance counselor is planning schedules for 200 students. 151 want to take Spanish and 49 want to take Latin. 19 say they want to take both. Display this information on the Venn Diagram.



TTL 200

8) What's the probability that a student studies at least one subject? P(SL) $\frac{132+19+30}{200} = \frac{181}{200}$

9) What's the probability that a student studies exactly one subject? $\frac{132+30}{200} = \frac{81}{100}$

10) What's the probability that a student studies neither subject? P(SL)' $\frac{19}{200}$

11) What's the probability that a student studied Spanish if it is known that he, she studies Latin? $\frac{19}{49}$

1,5, 5,1
2,4, 4,2
3,3

12) If you roll two die, find:

P(Odd number or a number greater than 8)

$\frac{3}{6} + 0 = \frac{3}{6} = \frac{1}{2}$

13) If you roll two die, find:

P(Doubles or a sum of 6)

$\frac{6}{36} + \frac{5}{36} = \frac{11}{36}$